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Gauge Symmetry and Supersymmetric Two-Particle Problem

R. P. Zaikov[1]

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An analogy between the removal of nonphysical relative time (or relative energy) in the supersymmetric two-particle problem and the account of local gauge invariance in supersymmetric quantum field theory is discussed. A group of gauge transformations for the Bethe-Salpeter amplitudes is suggested, the invariants of which are the relativistic three-dimensional (quasipotential) wave functions in the Logunov-Tavkhelidze approach. Subsidiary conditions imposed on the Bethe-Salpeter amplitudes in the Todorov approach are shown to be equivalent to appropriate gauge fixing.

The Bethe-Salpeter (BS) equations (Bethe and Salpeter, 1951) are a basic tool in investigations of the two-particle processes and bound-state problems in quantum field theory. However, a probabilistic interpretation of the BS amplitude is not possible because of the indefiniteness of the norm. This difficulty is solved by various modifications of the quasipotential approach (Logunov and Tavkhelidze, 1963; Kadyshevsky, 1968; Bogolubov, 1970; Todorov, 1971; Faustov, 1973; Garsevanishvili *et al.*, 1985), that is, by essentially three-dimensional formulations of the bound-state problem. The relative time or the relative energy is eliminated and the quasipotential wave functions are determined by the BS amplitudes on a certain space-time surface. The latter approach was generalized in Zaikov (1983, 1985) for the supersymmetric case also. The supersymmetric version of the Bethe-Salpeter equation was given earlier in the paper of Delburgo and Jervis (1974).

Similar difficulties with the nonphysical degrees of freedom are characteristic also of the gauge field theories. There the problem can be resolved in two different ways using the local gauge symmetry: First, one can formulate the theory in terms of gauge-invariant quantities, and second, an appropriate gauge can be fixed and thus one of the components of the

[1]Institute of Nuclear Research and Nuclear Energy, 1784 Sofia, Bulgaria.

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gauge field is eliminated. It is the exact gauge symmetry that justifies the additional conditions and guarantees the independence of the result on their choice.

In Koudinov and Zaikov (1982) a symmetry of the relativistic two-particle amplitude was studied and it was shown that the quasipotential operation of "equal-times" and also the subsidiary conditions on the wave function can be understood as implications of the corresponding invariance in complete analogy with the gauge theories. In the present article the results of the paper of Koudinov and Zaikov (1982) are generalized for the supersymmetric case for which the three-dimensional equations were written (Zaikov, 1983, 1985).

Let us consider the supersymmetric two-particle BS amplitude (Zaikov, 1983, 1985):

$$\Psi_{\mathscr{P}}(X, x; \theta_1, \theta_2) = \langle 0|\, T(\Phi^+(X + \mu_2 x, \theta_1)\Phi(X - \mu_1 x, \theta_2))|\mathscr{P}, j, j_3\rangle \quad (1)$$

Here $X = \mu_1 x_1 + \mu_2 x_2$ and $x = x_1 - x_2$ denote the center-of-mass and the relative coordinates, respectively, and $\mathscr{P}$ is the total momentum. For free particles the parameters μ_1 and μ_2 are easily found in terms of their masses:

$$\mu_1 = m_1/(m_1 + m_2), \qquad \mu_2 = m_2/(m_1 + m_2)$$

while for the interacting particles the only restriction on the values of $\mu_{1,2}$ is

$$\mu_1 + \mu_2 = 1$$

Thus, the position of the center of mass on the line between x_1 and x_2 is not defined and we consider the group of transformations with parameter λ:

$$X'_\mu = X_\mu + \lambda x_\mu$$

The BS amplitude then is transformed as follows:

$$\Psi'_{\mathscr{P}}(X, x; \theta_1, \theta_2) = \Psi_{\mathscr{P}}(X + \lambda x, x; \theta_1, \theta_2) = \exp(i\lambda x \mathscr{P})\, \Psi(X, x; \theta_1, \theta_2) \quad (2)$$

which looks like the $U(1)$ gauge transformation. In the ordinary case the latter is evident (see Koudinov and Zaikov, 1982). Here I will show that the same is true in the supersymmetric case also. I restrict consideration to the chiral superfields $\Phi^+(x, \theta)$ and $\Phi^-(x, \theta)$, where + (−) denote right (left) chiral superfield. Then the BS amplitude can be written in the following form (Zaikov, 1983, 1985):

$$\Psi_{\mathscr{P}}(X, x; \theta_1, \theta_2) = \begin{pmatrix} \Psi^{++}_{\mathscr{P}}(X, x; \theta_1, \theta_2) \\ \Psi^{-+}_{\mathscr{P}}(X, x; \theta_1, \theta_2) \\ \Psi^{+-}_{\mathscr{P}}(X, x; \theta_1, \theta_2) \\ \Psi^{--}_{\mathscr{P}}(X, x; \theta_1, \theta_2) \end{pmatrix} \quad (3)$$

Here $\Psi_{\mathscr{P}}^{ab}$ $(a, b=+,-)$ are the BS amplitudes (1) satisfying the chirality conditions:

$$\mathscr{D}_1^{-(+)}\Psi_{\mathscr{P}}^{+(-),b}=0, \qquad \mathscr{D}_2^{-(+)}\Psi_{\mathscr{P}}^{a,+(-)}=0 \qquad (a, b=+,-) \tag{4}$$

where

$$(\mathscr{D}_j^+)_a=\partial/\partial\theta_j^a, \qquad (\mathscr{D}_j^-)_a=\partial/\partial\theta_j^a \qquad (j=1,2)$$

are spinor covariant derivatives for chiral fields.

Then from (4) it follows that (2) are gauge transformations because they do not change the chirality of the BS amplitudes (3) (West, 1983). Now, following Koudinov and Zaikov (1982), I show that the supersymmetric version of the relativistic three-dimensional approach can be expressed in terms of gauge-invariant [with respect to the transformations (2)] quantities [the Logunov–Tavkhelidze approach (Zaikov, 1983)] or by gauge fixing [the Todorov approach (Zaikov, 1985)].

Let us start with the investigation of the "gauge" invariant quantities. Transcribing equation (2) in the momentum space, one obtains

$$\Psi_{\mathscr{P}}(p;\theta_1,\theta_2)\to\Psi_{\mathscr{P}}(p+\lambda\mathscr{P};\theta_1,\theta_2) \tag{5}$$

where $\Psi_{\mathscr{P}}(p;\theta_1,\theta_2)$ is the Fourier transform of BS amplitude (1) with respect to the relative coordinate:

$$\Psi_{\mathscr{P}}(X,x;\theta_1,\theta_2)=\exp(i\mathscr{P}X)\,\Psi_{\mathscr{P}}(x;\theta_1,\theta_2)$$

$$\Psi_{\mathscr{P}}(x;\theta_1,\theta_2)=\int d^4p\,\exp(ipx)\,\Psi_{\mathscr{P}};\theta_1,\theta_2)$$

The linear invariant of the "gauge" transformation (5) is

$$\Psi_{\mathscr{P}}(p_\perp;\theta_1,\theta_2)=\int dp_\parallel\,\Psi(p;\theta_1,\theta_2); \qquad \mathscr{P}p_\perp=0$$

which in the coordinate space corresponds to the "one-time" quasipotential wave function

$$\Psi_{\mathscr{P}}(x_\perp;\theta_1,\theta_2)=\int dx_\parallel\,\delta[x_\parallel+\varepsilon\gamma_\parallel(\theta_1-\theta_2)]\Psi_{\mathscr{P}}(x;\theta_1,\theta_2) \tag{6}$$

where

$$\xi_\parallel=n^\mu\xi_\mu=\mathscr{P}^\mu/\sqrt{\mathscr{P}^2}\,\xi_\mu, \qquad \xi_\perp^\mu=(g^{\mu\nu}+\mathscr{P}^\mu\mathscr{P}^\nu/\mathscr{P}^2)\xi_\nu$$

Here two particular cases are of interest. In the rest frame (in this frame the fermionic sector is also fixed) we get the Logunov–Tavkhelidze wave functions (Logunov and Tavkhelidze, 1963; Zaikov, 1983):

$$\Psi_E(x;\theta_1,\theta_2)=\int dx_0\,\delta(x_0)\Psi_E(x_0,x;\theta_1,\theta_2)$$

$$=\Psi_E(0,x;\theta_1,\theta_2)$$

$$\Psi_E(p;\theta_1,\theta_2)=\int dp_0\,\Psi(p_0,p;\theta_1,\theta_2)$$

If $\mathscr{P}$ is lightlike, one obtains the "one-time" wave function on the light cone (Garsevanishvili *et al.*, 1985; Zaikov, 1983):

$$\Psi_{\mathscr{P}_+}(x_-,x_\perp;\theta_1,\theta_2)=\int dx_+\,\delta(x_+)\Psi_{\mathscr{P}_+}(x_+,x_-,x_\perp;\theta_1,\theta_2)$$

$$\Psi_{\mathscr{P}_+}(p_-,p_\perp;\theta_1,\theta_2)=\int dp_-\,\Psi_{\mathscr{P}_+}(p_+,p_-,p_\perp;\theta_1,\theta_2)$$

where

$$\mathscr{P}_+=(\mathscr{P}_0+\mathscr{P}_3)/2,\qquad \mathscr{P}_-=(\mathscr{P}_0-\mathscr{P}_3)/2=0,\qquad \mathscr{P}_\perp=(\mathscr{P}_1,\mathscr{P}_2)=0$$

The second way to get rid of nonphysical "gauge" degrees of freedom is to fix an appropriate "gauge," that is, to impose on the BS amplitude some subsidiary condition. One of them is the Markov–Yukawa condition (Markov, 1940; Yukawa, 1950):

$$(\mathscr{P}p)\Psi(p;\theta_1,\theta_2)=0 \tag{7}$$

It does fix the "gauge," since if the BS amplitude $\Psi_{\mathscr{P}}(p;\theta_1,\theta_2)$ satisfies (7), then the transformed amplitude $\Psi'_{\mathscr{P}}(p;\theta_1,\theta_2)=\Psi_{\mathscr{P}}(p+\lambda\mathscr{P};\theta_1,\theta_2)$ satisfies (7) only for $\lambda=0$. Note that (7) is invariant with respect to the supertransformations, which follows from the commutativity of the momentum operator with the odd generators of the supersymmetry. In the coordinate space the condition (7) has the form

$$\partial^2\Psi_{\mathscr{P}}(X,x;\theta_1,\theta_2)/\partial X^\mu\,\partial x_\mu=0$$

and resembles the Lorentz gauge

$$\partial\Psi_\mu(X,x;\theta_1,\theta_2)/\partial X_\mu=0$$

if the notation

$$\Psi_\mu(X,x;\theta_1,\theta_2)=\partial\Psi_{\mathscr{P}}(X,x;\theta_1\Phi^l,\theta_2)/\partial x^\mu$$

is introduced and $\Psi_\mu(X,x;\theta_1,\theta_2)$ is regarded as an analog of the vector gauge superfield.

Any function satisfying equation (7) can be represented in the form

$$\Psi_{\mathscr{P}}(p;\, \theta_1, \theta_2) = \delta(p_{\|})\Psi_{\mathscr{P}}(p_{\perp};\, \theta_1, \theta_2)$$

where $p_{\|}$ and $p_{\perp}$ are given by (6).

Finally, note that the corresponding supersymmetric three-dimensional equations were written down in Zaikov (1983, 1985). Also, in the supersymmetric case the Todorov approach is more convenient because of the invariance of the subsidiary condition (7) with respect to the supertransformations.

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